1220: Energy of an electron confined in a small space

(From the uncertainty relation, we can conclude that particles confined in a small space have a large kinetic energy. This is the cause of resonance between unsaturated bonds in organic chemistry.)

Key words: Relationship between kinetic energy and momentum ambiguity; uncertainty relation; kinetic energy of an electron confined in space

When an electron (or any particle in general) is confined in a small space, things that are beyond common sense can happen. To make it easier to understand, let's look at an electron moving in one-dimensional space (on the *x*-axis).

The kinetic energy (T) of an electron with mass m and speed u is $(1/2)mu^2$. Rewriting this in terms of momentum ($p = mu$), we get $(1/2m)p^2$. Let's use "momentum ambiguity" (Δp) to find the kinetic energy.

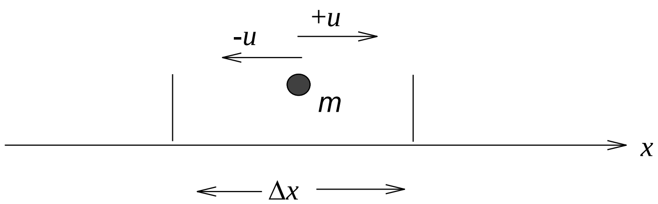

Figure 1. A particle of mass *m* moving through an area Δx.

Let the range of motion be Δx, and the momentum includes ambiguity Δp (if the correct value is p, then it is in the range of $p \pm \Delta p$). The correct value can be found by averaging the momentum including the ambiguity. Interestingly, a particle has the same probability of moving to the right and the left, so the momentum averages out to 0 ($p_{aver} = 0$).

$$p^2 = (p_{aver} \pm \Delta p)^2 = (\Delta p)^2$$

Using this relationship,

$$T = \frac{1}{2m}(\Delta p)^2$$

We introduce the uncertainty principle into this equation.

$$\Delta p \cdot \Delta x \geq \frac{\hbar}{2} \qquad \Rightarrow \qquad \Delta p \geq \frac{\hbar}{2\Delta x}$$

$$T \geq \frac{\hbar^2}{8m(\Delta x)^2}$$

The above equation shows that narrowing the range of motion of a particle increases its kinetic energy ($\Delta x \to 0$, $T \to \infty$). In other words, confining an electron to a small space requires a lot of energy. Conversely, electrons confined in a small space have a lot of energy, and this fact is very important in organic chemistry.